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Kelvin–Helmholtz instability in a compressible plasma with a magnetic field and velocity shear

M. G. Gudkov and O. A. Troshichev

Arctic and Antarctic Research Institute, St. Petersburg, Russia

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Abstract—The development of the Kelvin–Helmholtz instability (KHI) in a compressible plasma containing a magnetic field is studied for a finite thick layer with a velocity shear in a linear two-dimensional MHD approximation. Approximate analytical expressions for the phase speed, growth rate and wavenumber of the fastest growing unstable mode are derived for the velocity profile across the layer with the velocity shear having a sudden onset, termed a sharp elbow. The Dispersion equation is obtained from boundary conditions on the sharp elbow where the logarithmic derivative of the total pressure (or normal component of velocity) plays the main role.

The KHI, driven by velocity shear, is considered as a possible generator of plasma oscillations in magnetospheric regions such as at the boundary between the inner plasma sheet and the plasmasphere, the boundary between the plasma sheet and the tail lobes and the boundary between the plasma sheet and the magnetopause. Analytical expressions for the phase speed, growth rate, period and wavenumber of the growing unstable mode obtained with some simplifying assumptions leads to values similar to the results of numerical analyses.

1. INTRODUCTION

Periodic auroral forms are often observed at the borders of the auroral oval as wave-like structures with a certain periodicity and drifting along the border. The wave-like structures are detected in the evening sector at the oval's equatorward border, where large-amplitude undulations of the diffuse aurora boundary are seen (Lui *et al.*, 1982). The omega bands are typically observed in the morning sector of the poleward oval during auroral substorms. Discrete auroral arcs with a fan-shaped configuration occur in the cleft region (Lundin and Evans, 1985; Meng and Lundin, 1986). Using UV auroral images from Viking spacecraft, Lui *et al.* (1989) found spatially periodic bright spots in the afternoon sector which they described as 'beads on a string'.

Lui *et al.* (1982) first suggested the Kelvin–Helmholtz instability as the possible mechanism of development of wave-like processes at the inner edge of the plasma sheet in the evening sector where the sunward convecting plasma encounters the corotating plasmasphere. The KHI, driven by velocity shear, is usually regarded as a possible mechanism to generate a surface wave in the plasma sheet boundary layer (Lyons and Fennel, 1986; Lui *et al.*, 1987; Bythrow *et al.*, 1986; Bythrow *et al.*, 1987; Troshichev, 1991). The KHI is examined in the low-latitude boundary layer as one of alternatives to explain periodic variations in the magnetic field, energetic particle fluxes, and auroral forms on the dayside of the magnetosphere (Bythrow *et al.*, 1986; Lui *et al.*, 1989; Potemra *et al.*, 1992).

While reference to the KHI is in common use in treatments of wave-like auroral forms, a detailed consideration of the instability mechanism has not yet been performed. There are a number of theoretical studies (Miura and Pritchett, 1982; Pritchett and Coroniti, 1984; Thompson, 1983), as well as numerical calculations (Wu, 1986; Wei *et al.*, 1990; Mesensev, 1991; Morozov and Mishin, 1981; Miura, 1987Miura, 1982; Yamamoto *et al.*, 1991Yamamoto *et al.*, 1993), of the K–H instability, but analytical expressions for the phase speed, growth rate, and wavenumber of the fastest growing unstable mode for the case of an arbitrary velocity profile have not yet been obtained. Here we consider such a possibility a velocity shear profile with a sudden onset, termed a sharp elbow in the velocity profile. Other plasma parameters such as the magnetic field, sound speed and density can also be changed at this point.

The boundary conditions at the sharp elbow lead to the dispersion equation allowing us to define the period, the phase speed, and the growth rate of the oscillations. Derivation of the analytical expressions for these values is the aim of this study.

2. ANALYTICAL DEPENDENCE ON THE ARBITRARY VELOCITY PROFILE

The flow of compressible plasma in a magnetic field is examined using the linear two-dimensional MHD approximation. The wave is supposed to be plane, with the wave vector **k** lying in the plane of the flow, the x axis is directed along the wave vector, and the z axis is directed across the flow in the direction of the flow velocity increase. $\partial/\partial y = 0$ for all quantities. $\partial/\partial x = 0$ for the background quantities. The flow velocity is $U = (U_x, U_y, 0)$, the plasma density and magnetic field are constant ($\rho = \text{const}$, $H = \text{const}$), the sum of the gas and magnetic pressures is $p = \text{const}$, $H_z = 0$. All perturbed values are written in the form:

$$A \sim A(z)\exp(i(kx-\omega t+\varphi(z)))$$

Three components of the flow equation are used:

$$-i\omega\rho V_x + ik\rho U_x V_x + \rho U_x' V_z = -ikp + \frac{ik}{4\pi}H_x h_x, \tag{1}$$

$$-i\omega\rho V_y + \rho U_y' V_z + ik\rho U_x V_y = \frac{ik}{4\pi}H_x h_y, \tag{2}$$

$$-i\omega\rho V_z + ik\rho U_x V_z = -p' + \frac{ik}{4\pi}H_x h_z. \tag{3}$$

The continuity equation:

$$-i\omega\tilde{\rho} + ik\tilde{\rho}U_x + ik\rho V_x + \rho V_z' = 0 \tag{4}$$

and Maxwell's equations from which the components of the magnetic field are expressed as:

$$h_x = -\frac{iH_x}{k}\left(\frac{V_z}{\Omega}\right)', \tag{5}$$

$$h_y = \frac{iH_y(V_z'/k + iV_x)\Omega}{V_{ax}^2 - \Omega^2}, \tag{6}$$

$$h_z = -\frac{V_z H_x}{\Omega}, \tag{7}$$

where

$$\Omega = \frac{\omega}{k} - U_x,$$

$$p = C_s^2\tilde{\rho} + \frac{H_x h_x}{4\pi} + \frac{H_y h_y}{4\pi} \tag{8}$$

ρ is the density, C_s is the sound speed, H is the magnetic field, $\tilde{\rho}$ is the density perturbation amplitude, V is the velocity perturbation amplitude, U is the undisturbed velocity, V_a is the Alfven speed, and h is the magnetic field perturbation amplitude. A prime denotes the derivative with respect to z.

The velocity component V_z can be derived from (3) and (7):

$$V_z = \frac{ip\Omega}{k\rho(V_{ax}^2 - \Omega^2)}, \tag{9}$$

and V_y from (2):

$$V_y = \frac{4\pi\rho U_y' V_z - ikh_y H_x}{4\pi ik\rho\Omega}. \tag{10}$$

The density $\tilde{\rho}$ is expressed from (8), h_x and h_y are substituted there from (6) and (7). Inserting (8), (5) and (6) into (4), we obtain:

$$\frac{V_z'}{k} + iV_x = \frac{i}{\Omega}\left(\frac{p}{\rho} - \frac{V_{ax}^2}{ik}\left(\frac{V_z}{\Omega}\right)'\right)\Big/ f, \tag{11}$$

where

$$f = \frac{C_s^2}{\Omega^2} - \frac{V_{ay}^2}{V_{ax}^2 - \Omega^2}$$

Let us substitute V_x in the x-component of the flow equation.

$$k\rho\Omega\left[\frac{V_z'}{k} - \frac{i}{\Omega}\left(p/\rho - \frac{V_{ax}^2}{ik}\left(\frac{V_z}{\Omega}\right)'\right)\Big/ f\right] + \rho V_z U_x' = -ikp + \rho v_{ax}^2\left(\frac{V_z}{\Omega}\right)' \tag{12}$$

In this equation V_z is substituted from (9).

$$\left(\frac{V_z}{\Omega}\right)'\left[\frac{\Omega^2 f}{f-1} - V_{ax}^2\right] + \frac{ikp}{\rho} = 0, \tag{13}$$

By using (9) equation (13) resolves solely into V_z variables or solely into p variables.

$$\left(\frac{p'}{V_{ax}^2 - \Omega^2}\right)' + \frac{k^2 p}{G} = 0, \tag{14}$$

$$\left(\left(\frac{v_z}{\Omega}\right)' G\right)' + \frac{k^2(V_{ax}^2 - \Omega^2)}{\Omega}V_z = 0, \tag{15}$$

where

$$G = \frac{\Omega^2 f}{f-1} - V_{ax}^2.$$

In the simplest case of an incompressible medium without a magnetic field, equation (15) is simplified to Rayleigh's equation:

$$V_z'' + V_z U_x''/\Omega - k^2 V_z = 0. \tag{16}$$

Equation (14) is an analogue of the equation obtained by Miura and Pritchett (1982).

The analytical solution of these equations for an arbitrary velocity profile does not seem to be possible, but a general idea about the nature of the disturbance can be obtained considering the imaginary part of the logarithmic derivative of the total pressure or the transverse speed. This quantity is equal to a derivative of the oscillation phase for p (or V_z) with respect to z. Therefore, this imaginary part characterizes an eddy of the flow. It appears that this imaginary part at an arbitrary point z_o can be expressed through parameters of the oscillation and background plasma parameters at the point z_o and at some resonant points where equations (14)–(16) have singularities. Such expressions are helpful for understanding the physics of these phenomena.

In order to find $\Im m(p'/p)$ we introduce a new variable w:

$$p = w(V_{ax}^2 - \Omega^2)^{1/2}. \qquad (17)$$

then equation (14) loses the first derivative term.

$$w'' + w\left(\frac{k^2(C_s^2 V_{ax}^2 - C_f^2\Omega^2 + \Omega^4)}{C_f^2\left(U_x - \frac{\omega}{k} - V_{kx} - i\delta\right)\left(U_x - \frac{\omega}{k} + V_{kx} - i\delta\right)} + \frac{U_x''\Omega - (U_x')^2}{\left(V_{ax} - \frac{\omega}{k} + U_x - i\delta\right)\left(V_{ax} + \frac{\omega}{k} - U_x + i\delta\right)} - \frac{3\Omega^2 (U_x')^2}{\left(V_{ax} - \frac{\omega}{k} + U_x - i\delta\right)^2\left(V_{ax} + \frac{\omega}{k} - U_x + i\delta\right)^2}\right) = 0, \qquad (18)$$

where $C_f^2 = C_s^2 + V_a^2$ is the square of the fast magnetosonic speed along the magnetic field, and $V_{kx} = C_s V_{ax}/C_f$. The small quantity $i\delta$ is introduced here for contour integral evaluation are singularity points in conformity with Landau's rule: (when $t \to -\infty$ the amplitude of oscillation must tend to zero).

Assume that ϑ is a solution of the Lagrange conjugate equation which differs from (18) only by the sign of $i\delta$. Let us multiply equation (18) by ϑ, the conjugate equation multiply by w, and then subtract one from the other.

$$\int_{z_o}^{\infty} (w''\vartheta - \vartheta'' w)\,dz = (\vartheta w' - \vartheta' w)|_{z_o}^{\infty} = -\vartheta(z_o)w'(z_o) + \vartheta'(z_o)w(z_o) = -2\pi i \sum_{j=n}^{N} \mathrm{Res}\,\vartheta_j w_j (F_j - \bar{F}_j), \qquad (19)$$

where F is the multiplier of w in (18), $\bar{F}$ is the conjugate of F, N is the total number of residues, n is the number of the residue which is closest to the point z_o in the direction of increasing z.

$$\Im m\, p'/p = \Im m\, w'/w + \Im m\, \Omega U_x'/(V_{ax}^2 - \Omega^2). \qquad (20)$$

Considering (19) for the infinitely distant point, one can conclude that the total sum of residues is equal to zero in any case.

$$|w_b|^2 \frac{\pi k^2 V_{kx}^3}{2U_{xb}' C_f^2}\bigg|_{U_{xb} = \omega/k - V_{kx}} - |w_a|^2 \frac{\pi k^2 V_{kx}^3}{2U_{xa}' C_f^2}\bigg|_{U_{xa} = \omega/k + V_{kx}} - \left(\frac{\pi U_x''}{2U_x'} - \frac{\pi U_x}{4V_{ax}}\right)|w_c|^2|_{U_{xc} = \omega/k + V_{ax}} - \left(\frac{\pi U_x''}{2U_x'} - \frac{\pi U_x'}{4V_{ax}}\right)|w_d|^2|_{U_{xd} = \omega/k - V_{ax}} = 0. \qquad (21)$$

Integrating (15) as well as (14) we obtain nine resonant points: $U_x(z_j) - U_{ph} = 0$, $\pm V_{kx}$, $\pm V_{ax}$, $\pm((C_f^2 \pm C_f(C_f^2 - 4V_{kx}^2)^{1/2})/2)^{1/2}$ (U_{ph} is the phase speed). All these nine residues will not be written here since they are cumbersome and only the simplest case (16) will be considered. For an arbitrary point z_o we obtain:

$$\Im m\, V_z'/V_z|_{z_o} = -\pi \frac{U_x''(z_c)|V_z(z_c)|^2}{U_x'(z_c)|V_z(z_c)|^2}\bigg|_{U_x(z_c) = U_{ph}}. \qquad (22)$$

The matter "$|_{U_{xb} = w/k - V_{kx}}$" means that the expression "$|w_b|^2(\pi k^2 V_{kx}^3/2U_{xb}' C_f^2)$" must be taken in the point where $U_{xb} = w/k - V_{kx}$.

Considering the infinitely distant point one can see that $U_x''(z_c) = 0$. This means that the phase speed always coincides with the velocity flow at the elbow point for incompressible flow without a magnetic field. This is stronger than the well known Rayleigh theorem which simply declares the necessity of the existence of an elbow in the velocity profile for the development of oscillations.

Unfortunately, for an arbitrary velocity profile we can do nothing more. However, the obtained formulae turn out to be useful when considering an idealized profile with a sharp elbow that may exist in the magnetosphere. For example, the magnetic field at the magnetopause changes over a distance considerably less than the typical scale size over which the plasma flow charges. The plasma velocity begins to change immediately towards the boundary layer (Sergeev and Tsyganenko, 1980). According to this fact a discontinuity of the first derivative of the plasma velocity at the magnetopause can be supposed. A similar situation exists at the boundary between the plasmasheet and the tail lobes (Rajaram *et al.*, 1991).

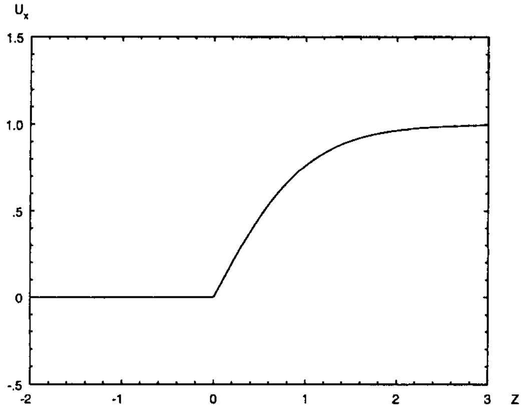

Fig. 1. The velocity profile with a sharp elbow; the direction of flow is perpendicular to the z-axis.

3. VELOCITY PROFILE WITH A SHARP ELBOW

It is known (Landau and Lifshits, 1988) that oscillations of the flow are concerned with the second derivative of the velocity. This can be seen from the eigenmode equation (e.g., Miura and Pritchett, 1982), which never has a solution satisfying the necessary boundary conditions when $U_x'' \equiv 0$. This means that areas with the maximum curvature of the profile are the main sources of oscillations. Let us examine the case when the velocity derivative changes suddenly at one point of the profile; the curvatures at other points are negligible (Fig. 1). The jump of the first derivative of the velocity occurs at this point. The surface wave will run along the boundary, i.e. along the surface of maximum curvature.

Usually the wave amplitude increases if the number of faster particles is larger than the number of slower particles at the resonant points: this depends on the sign of the second derivative of the velocity. The wave energy will be provided at the cost of flattening the profile curvature. Using this method we shall try to obtain the analytical dependance and to understand better the physics of the Kelvin–Helmholtz instability. Such a method of examination was first put forward by Phillips (1980) for ocean waves; this was later used for an incompressible medium containing a magnetic field in applications to problems of astrophysics (Vrobel and Kontorovich, 1982; Gestrin and Kontorovich, 1984); it was applied for the magnetosphere by Ptitsina and Gudkov, 1991.

The present study concerns a compressible medium with a magnetic field. We consider the case when the magnetic field, sound speed and density of medium are changed at the plane boundary where the first derivative of velocity U'_{xo} (quantities on the boundary will be denoted by '$_o$') has the jump.

Moreover, in order to obtain the dispersion equation it is necessary to suppose that the flow velocity on one side of the boundary is equal zero. All parameters on this side will be denoted by '$_z$', and, in the medium with flow velocity, by the index '$_1$'. The flow velocity on the boundary is continuous, so $U_{xo} = 0$.

A location where the scale of change of magnetic field in the the velocity shear layer is considerably less than that the flow velocity can be found in the magnetosphere. Rajaram *et al.* (1991) have considered the sharp jump of magnetic field on the edge of the layer of smoothly varying velocity shear, that is, in the low-latitude boundary layer. However, the solution has been sought numerically for the velocity profile in the form of hyperbolic tangents. Similar situations can be also realized at the magnetopause (Sergeev and Tsyganenko, 1980). Using the condition of the absence of the normal plasma flow across the boundary (the tangential discontinuity) we obtain: $V_{z1o} = V_{z2o}$. But for the dispersion equation the relationship between V'_{z1o} and V'_{z2o} is essential as well. For this purpose we shall seek the exact solution for V_{z2} using the fact that the plasma velocity is equal to zero on this side of the boundary. Instead of (15) the simple equation with constant coefficients is obtained:

$$V_z''\left(\frac{\Omega f}{f-1} - \frac{V_{ax}^2}{\Omega}\right) + \frac{k^2 V_z (V_{ax}^2 - \Omega^2)}{\Omega} = 0. \quad (23)$$

$$V_{z2} = V_{z2o} \exp\left(kz((\Omega^2 - V_{ax2}^2)/G)^{1/2}\right). \quad (24)$$

Therefore, the solution is found only for the case when the expression under the square root is positive. This means that:

$$(\mathrm{Re}\,\omega/k)^2 \in [0, V_{kx2}] \cup [x_1, x_2]$$

where

$$V_{kx2} = C_{s2} V_{ax2} / C_{f2}$$

is the cusp resonance speed in the direction of x, and

$$x_{1,2} = (C_{f2}^2 \pm (C_{f2}^4 - 4V_{kx2}^2 C_{f2}^2)^{1/2})/2$$

are the speeds of the fast and slow magnetosonic waves. Then the condition of total pressure balance: $p_{1o} = p_{2o}$, where p_{2o} is expressed from (13), can be used.

$$p_{1o} = -\frac{\rho_2}{ik}\left(\frac{V_{z2}}{\Omega}\right)'_o G_2. \quad (25)$$

Now we substitute the solution (24) for (V_{z2}/Ω), replace V_{z2o} with V_{z1o}, and substitute the expression for V_{z1o} from (9). The dispersion equation for the pressure variables is obtained as:

$$k\frac{\rho_1}{\rho_2}(V_{ax1}^2-\Omega^2) = -\left(\frac{p_1'}{p_1}\right)_o((\Omega^2-V_{ax2}^2)/G_2)^{1/2}G_2. \tag{26}$$

The dispersion equation for the V_z variables can be obtained in a similar way.

$$\rho_1\left(\frac{V_{z1}}{\Omega}\right)_o' G_1 = \rho_2 G_2^{1/2}k(\Omega^2-V_{ax2}^2)^{1/2}\frac{V_{z1o}}{\Omega}. \tag{27}$$

Quantities $(p_1'/p_1)_o$ or $(V_{z1}'/V_{z1})_o$ in the dispersion equations remain unknown.

The general scheme of solution is the following. Since we have obtained a dispersion equation of fourth order, it is preferable to reduce it to second order using assumptions about the relationship between the main plasma parameters. These relationships are different for various areas of the magnetosphere. Expressions for the phase speed and growth rate can be obtained more or less easily from the second order equation with additional assumptions. Moreover, as a rule, the imaginary part of the logarithmic derivative of the pressure (or transverse speed) plays the main role in determining the growth rate. These derivatives can be expressed in terms of parameters of the oscillation and background plasma at the resonant points, as well as at the boundary. For example:

$$\Im m\left.\frac{w'}{w}\right|_o = \pi\sum_{j=1}^{N}\text{Res}\frac{|w_j|^2}{|w_o|^2}\cdot(F_j-\bar{F}_j). \tag{28}$$

Here the quantities

$$|w_j|^2/|w_o|^2$$

are unknown and they can be estimated approximately resolving the differential equation (18) with an asymptotic consideration of the singularities. In this case it is useful to take into account the infinitesimal nature of second derivative of the flow velocity in the area of integration excluding the elbow. The real part $(p_1'/p_1)_o$ (or $(V_{z1}'/V_{z1})_o$) has been found from the approximate solution in the region near the boundary.

The wavenumber of the fastest growing unstable mode usually can be found by differentiating the growth rate with respect to k.

An illustration of the results for examples of the simplest velocity profile and the realization of this scheme for concrete conditions in the magnetosphere will now be presented.

4. SOLUTION FOR LINEAR VELOCITY PROFILE

Let us consider the plane flow of incompressible fluid without a magnetic field. The velocity in the layer of thickness z is changed linearly from 0 to 2 and is constant outside this region (Fig. 2). Such a flow is described by equation (16). The surface wave, running along each elbow, can be determined from the dispersion equation (27). In this case the dispersion equation is written as:

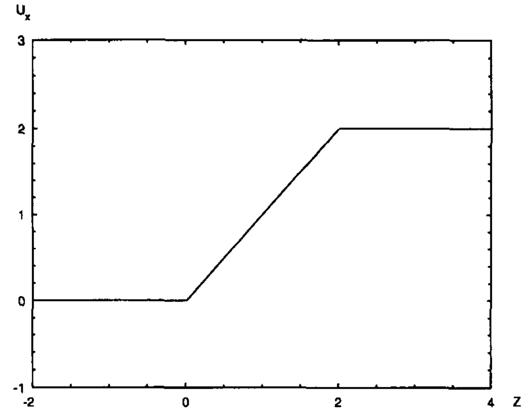

Fig. 2. The linearly-varying velocity profile; the direction of flow is perpendicular to the z-axis.

$$\Omega = \Delta U_x'/(k-(V_z'/V_z)_o) = U_f/L/(k-(V_z'/V_z)_o). \tag{29}$$

$(V_z'/V_z)_o$ is found from Rayleigh equation (16).

$$\text{Re}(V_z'/V_z)_o = k(\exp(-2k)-1)/(1+\exp(-2k)) \tag{30}$$

$$U_{ph} = (1+\exp(-2k))/(2k) \tag{31}$$

$U_{ph} = 0$ for $k = 0.64$, as it should be for the given profile.

$$\gamma = \Im m(\omega) = \frac{k\Im m(V_z'/V_z)_o}{(1/U_{ph})^2+(\Im m(V_z'/V_z)^2}. \tag{32}$$

For the Rayleigh equation we shall consider:

$$U_x''(z) = \delta(0)-\delta(2),$$

where $\delta(z)$ is the Dirac delta-function. Integrating the Rayleigh equation in the same manner as we did when equation (19) was derived, we obtain:

$$\Im m(V_z'/V_z)_o = \frac{\dfrac{\exp(-4k)\gamma/k}{(2-U_{ph})^2+\gamma^2/k^2}+\dfrac{\gamma/k}{U_{ph}^2+\gamma^2/k^2}}{1+\exp(-4k)}. \tag{33}$$

The total solution for the system, including equations (32) and (33), is too bulky. Let us study some special cases. It may be seen that the growth rate is equal to 0 for $U_{ph} = 1$ ($k = 0.64$), reaches a maximum close to $U_{ph} \cong 2(k \cong 0.4)$ and again goes to 0 for $k = 0$. Let us estimate the maximum growth rate.

$$\Im m(V_Z'/V_Z)_o|_{U_{ph}=2} \cong \frac{1}{2}\exp(-4k), \quad (34)$$

$$\gamma \cong 0.4\frac{1}{2}\exp(-4\times 0.4)/(1/2)^2 = 0.16. \quad (35)$$

The growth rate is maximal when the phase speed of the surface wave generated by the first elbow coincides with the flow speed in the region of the second elbow. The sum of these two oppositely travelling plane waves produces a standing wave relative to the centre of symmetry.

Numerical analyses, carried out by Ong and Roderick (1972) and Miura and Pritchett (1982), show that the maximal value of γ is equal to 0.18 for $k = 0.45$. In the case of a linear profile, Ong and Roderick, 1972 obtained γ equal to zero for the same wave number ($k = 0.64$).

5. BOUNDARY BETWEEN PLASMA SHEET AND TAIL LOBES

We assume for this case that the width of the current sheet, separating the plasma sheet (medium 1) from the magnetospheric tail (medium 2) is much less than the width of the layer where the plasma velocity changes, this layer being located within the plasma sheet close to the current sheet. In this case the magnetic field is parallel to the flow. Taking into account the observed values of the fast magnetosonic speed $C_{f1} = 1000$ km/s, and Alfven speed $V_{a1} = 900$ km/s, we obtain the sonic speed $C_{s1} = 435$ km/s, and the speed of cusp resonance $V_{k1} = 390$ km/s. We assume also that the plasma density is different in both media ($\rho_1 = 10\ \rho_2$), and $C_{s1} = C_{s2}$. We shall consider only the waves propagating along the plasma flow because numerical calculations show a maximum growth rate for such waves (Miura and Pritchett, 1982). In our case when the Alfven poles are absent ($V_{a1} > U_{max}$) the following condition is necessary for the development of oscillations:

$$\Omega_o + V_{kx1} < U_{max}.$$

Here $\Omega = \omega/k - U_x$, U is the plasma flow velocity; index 'o' is taken for values on the boundary. To simplify the dispersion equation we can then assume:

$$V_{ax1}^2 \gg \Omega_o^2.$$

We shall suppose also that

$$\Omega_o^2 \gg V_{ax2}^2.$$

Then the dispersion equation of fourth order (26) can be reduced to a quadratic equation in Ω^2 for the pressure variables:

$$(kV_{ax1}^2\rho_1/\rho_2)^2(C_{s2}^2 - \Omega^2) = \left(-\frac{p_1'}{p_1}\right)^2\Omega^4 C_{s2}^2. \quad (36)$$

Solving this equation we obtain expressions for the phase speed and growth rate.

$$U_{ph} = C_{s2} \quad (37)$$

$$\gamma = -\frac{C_{s2}^5\rho_2^2}{kV_{ax1}^4\rho_1^2}\Im m(p'/p)_o \operatorname{Re}(p'/p)_o. \quad (38)$$

Let us introduce the value $\Im m(w'/w)$ where:

$$w = p/(V_{ax}^2 - \Omega^2)^{1/2}. \quad (39)$$

In this case only two poles of the cusp resonance exist.

$$\Im m(w'/w)_o = \frac{|w_b|^2}{|w_o|^2}\frac{\pi k^2 V_{kx1}^2}{2U_{xb}'C_{f1}^2}\bigg|_{U_{xb}-\omega/k-V_{kx1}} - \frac{|w_a|^2}{|w_o|^2}\frac{\pi k^2 V_{kx1}^2}{2U_{xa}'C_{f1}^2}\bigg|_{U_{xa}=\omega/k+V_{kx1}}. \quad (40)$$

The ratio

$$|w_i|^2/|w_o|^2$$

may be obtained from (18). At first, we shall find a solution in the regions located far from the poles $\pm V_{kx1}$ by taking into account that $V_{ax1} \gg U_x$, C_{s1}. A simple equation is obtained.

$$w'' - k^2 w = 0. \quad (41)$$

To find the asymptotic solution close to the poles we expand the denominators in equation (18) into a series.

$$\Omega - V_{kx1} = -zU_x'. \quad (42)$$

$$w'' + w\Phi^2/z = 0, \Phi^2 = \frac{k^2(C_{s1}^2 - V_{kx1}^2)(V_{ax1}^2 - V_{kx1}^2)}{2V_{kx1}U_x'C_{f1}^2}. \quad (43)$$

After substituting:

$$w_1 = z^{1/2}T(2\Phi z^{1/2}) \quad (44)$$

the equation (43) is reduced to a first-order Bessel equation for positive z and to a modified first-order differential Bessel equation for negative z. The Bessel functions J_1 and I_1 are selected taking into account the boundary condition, that the value w would be limited at the zero point. The general solution of equation (43) is found from a partial solution according to the formula:

$$w(z) = w_1(z)\left(c_1 + c_2 \int \frac{\mathrm{d}z}{w_1^2(z)}\right). \quad (45)$$

$$\int \frac{\mathrm{d}z}{z^{1/2} J_1^2(2\Phi z^{1/2})} = 2 \int \frac{\mathrm{d}\tilde{z}}{\tilde{z} J_1^2(\tilde{z})} \quad (46)$$

Considering $J_1 = -J'_o$,

$$\int \frac{\mathrm{d}\tilde{z}}{\tilde{z}(J'_o)^2} = \int \frac{\mathrm{d}(\ln \tilde{z})}{(J'_o)^2} \quad (47)$$

$$(1/(\tilde{z}J_oJ_1))' = 1/(\tilde{z}J_1^2) - 1/(\tilde{z}J_o^2). \quad (48)$$

Then, in the region $\tilde{z} \cong 0$, we obtain roughly:

$$\int [(1/(\tilde{z}J_oJ_1))' + 1/(\tilde{z}J_o^2)]\,\mathrm{d}\tilde{z} \cong 1/(\tilde{z}J_oJ_1) + \frac{\ln \tilde{z}}{J_o^2}, \quad (49)$$

$$w(z) = c_1 z^{1/2} J_1(2\Phi z^{1/2}) + c_2/J_o(2\Phi z^{1/2})$$
$$+ c_2 2\Phi z^{1/2} J_1(2\Phi z^{1/2}) \ln(2\Phi z^{1/2})/J_o^2(2\Phi z^{1/2})$$
$$\cong c_2/J_o(2\Phi z^{1/2}). \quad (50)$$

Here we take $c_1 = 0$ for simplicity because, in other cases, the solution will represent a funnel with steep edges which make no physical sense. In reality, the value $2\Phi z^{1/2}$ varies from 0 to 0.01. Within this range we obtain negligible changes for $J_o = 1 - 0.999$, and for $I_o = 1 - 1.001$. Therefore, the value w varies negligibly in the vicinity of the poles and we can derive the approximate solution for the whole region.

$$w = w_o \exp(-kz). \quad (51)$$

Substituting this solution into (40), we obtain an expression for the growth rate. For large values of V_{ax1}:

$$\Im m(p'/p)_o = \Im m(w'/w)_o + \Im m \Omega_o U'_{xo}/(V_{ax}^2 - \Omega_o^2)$$
$$\cong \Im m(w'/w)_o. \quad (52)$$

$$\Im m(p'/p)_o = \frac{\pi}{2} \frac{k^2 V_{kx1}^3}{C_{f1}^2} \left(\frac{\exp(-2kz_\alpha)}{U'_{x\alpha}} - \frac{\exp(-2kz_\beta)}{U'_{x\beta}} \right); \quad (53)$$

$U_{x\alpha} = U_{ph} + V_{kx1}$; $U_{x\beta} = U_{ph} + V_{kx1}$.

These values we shall substitute into formula (28).

The growth rate γ can be presented as the sum of two terms: $\gamma = \gamma_\alpha + \gamma_\beta$, where

$$\gamma_\alpha = \frac{\pi}{2} (\rho_2/\rho_1)^2 \frac{C_{s2}^5 V_{kx1}^3}{V_{ax1}^4 C_{f1}^2} \frac{k^2 \exp(-2kz_\alpha)}{U'_{x\alpha}}; \quad (54)$$

$$\gamma_\beta = \frac{\pi}{2} (\rho_2/\rho_1)^2 \frac{C_{s2}^5 V_{kx1}^3}{V_{ax1}^4 C_{f1}^2} \frac{k^2 \exp(-2kz_\beta)}{U'_{x\beta}} \quad (55)$$

Both terms have a maximum at $k_i = 1/(2z_i)$. The term γ_α increases the perturbation and it depends on the velocity parameters in the faster region. The term γ_β decreases the perturbation and depends on speed parameters in the slower region.

Taking the known values $U'_{x\alpha} = 0.1\ \mathrm{s}^{-1}$, $U'_{x\beta} = 1\ \mathrm{s}^{-1}$, $z_\alpha = 4000$ km, $z_\beta = 2000$ km, we can calculate the maximum values γ_a (for $k = 1/(2z_\alpha)$) and γ_β (for $k = 1/(2z_\beta)$). The maximum γ_β turns out to be ten times greater than the maximum γ_β, i.e., the velocity profile at point z_α is less steep than at the point z_β. Other estimated values are the following: $\gamma_{max} \cong \gamma_{max}(z_\alpha) \cong 0.14\ \mathrm{s}^{-1}$, $U_{ph} = 300$ km/s, $T = 3$ min. The typical growth time is $\tau = 1/\gamma = 7$ s. Although selection of the value U_x' is rather arbitrary, the estimated values of the growth rate appear to be correct.

These disturbances would map to the poleward boundary of the diffuse auroral zone near local midnight. Auroral oscillations, related to these disturbances can move eastward (as well as westward) as a consequence of the electron drift under the influence of the electric field, and gradients of pressure and magnetic field. The drift speed at ionospheric heights is about 5 km/s and the wavelength is about 500 km.

Corresponding wave-like oscillations of the poleward boundary of diffuse precipitation are observed during substorms in the morning sector of the auroral oval, with the name omega-bands. It is likely that the auroral westward travelling surge is a manifestation of the KHI which develops near midnight and drifting westward. Contrasts between pre-midnight and post-midnight oscillations can be caused by differences in the source of drift.

6. BOUNDARY BETWEEN INNER PLASMA SHEET AND PLASMASPHERE

In the evening sector, plasmasheet particles convect sunward whereas particles in the plasmasphere corotate in the opposite direction. The velocity difference across the shear layer is about 100 km/s here. As before we suggest that the velocity is zero in one medium, i.e. the velocity elbow is located at the edge of the velocity profile. The magnetic field has the same direction on both sides of the boundary and it is perpendicular to the plasma flow. In this case the dispersion equation can be written as:

$$\rho_1(V_{z1}/\Omega)'_o \frac{C_{f1}^2}{C_{f1}^2-\Omega_o^2} = \rho_2 \frac{C_{f2}}{C_{f2}^2-\Omega_o^2} \times k\frac{V_{z1o}}{\Omega_o}(C_{f2}^2-\Omega_o^2)^{1/2}. \quad (56)$$

The following simplification can be used:

$$C_{f1}^2, C_{f2}^2 \gg \Omega_o^2.$$

Then:

$$\Omega_o = U'_{xo}/(k\rho_2/\rho_1-(V'_{z1}/V_{z1})_o), \quad (57)$$

$$U_{ph} = \frac{U'_{xo}(k\rho_2/\rho_1-\mathrm{Re}\,(V'_{z1}/V_{z1})_o)}{|k\rho_2/\rho_1-(V'_{z1}/V_{z1})_o|^2}, \quad (58)$$

$$\gamma = \frac{kU'_{xo}\Im m(V'_{z1}/V_{z1})_o}{|k\rho_2/\rho_1-(V'_{z1}/V_{z1})_o|^2}. \quad (59)$$

The differential equation (15) will be reduced to the Rayleigh equation (16). The expression for the imaginary term of the logarithmic derivative of the transverse speed (22) is obtained by integrating (16). This value is equal to the derivative of the oscillation phase V_z with respect to z and, therefore, it describes a velocity vortex. The vortex centre is at the point z_c where $U_x = U_{ph}$.

Let us examine the behaviour V_{z1} at the point z_c.

$$V''_{z1} + \frac{U''_x V_{z1}}{U'_x z} = 0 \quad (60)$$

This equation is analogous to (42) if $\Phi^2 = |U''_x/U'_x|$. Here $2\Phi z^{1/2}$ depends on the curvature of the original profile at the point z_c. Assuming the ratio $|U''_x(z_c)/U'_x(z_c)|$ to be less than $1/L$, we can use the approximate solution (41) for the whole region. Substituting this solution into (22), expressions for the phase speed and growth rate can be obtained.

$$U_{ph} = \rho_1/(\rho_1+\rho_2)U'_{xo}/k, \quad (61)$$

$$\gamma = -\frac{U'_{xo}}{k}\frac{\pi\rho_1}{\rho_1+\rho_2}\frac{U''(z_c)}{U'(z_c)}\exp(-2kz_c) \quad (62)$$

We can see that the growth rate decreases monotonically when k increases. This means that the growth rate reaches a maximum value for the minimum wavenumber, which may be defined from (62) for the maximum phase speed.

$$k = \frac{\rho_1}{\rho_1+\rho_2}U'_{xo}/U_{ph} = \frac{\rho_1}{\rho_1+\rho_2}U'_{xo}/U_{xmax}. \quad (63)$$

Maximum values U_{xmax} are far from the boundary where z_c becomes maximum. Thus, we obtain the phase speed equal to the maximum velocity of the flow. The wavenumber for $\rho_1 = \rho_2$ turns out to be the following:

$$k = \frac{U'_{xo}}{2U_{xmax}} \quad (64)$$

where U'_{xo} is the derivative of the plasma flow velocity with respect to z on the boundary. For

$$U'_{xo} = U_{xmax}/L \cong 0.02\ \mathrm{s}^{-1}$$

we have $\lambda \cong 6\ 10^4$ km, $T \cong 10$ min. The wavelength in the ionosphere would be 200 times smaller, i.e. about 300 km.

According to Lui *et al.* (1982), observed oscillations of the equatorward boundary of the diffuse auroral oval show wave lengths in range 200–900 km. To provide this wavelength for a given phase velocity 100 km/s, the value U'_{xo} must change in the range 0.007–0.03 s^{-1} which seems to be feasible. The growth rate for $L \sim 1\ R_E$ is $\gamma \sim 0.05\ \mathrm{s}^{-1}$ and a typical growth time would be 20 s.

We can see from (64) that the growth rate is proportional to U'_{xo}, which is a value of the velocity derivative jump on the boundary characterizing the significance of the surface. Since the wavelength and period are inversely proportional to U'_{xo}, disturbances with the least wavelength and period will have the largest amplitude among the different disturbances.

7. MAGNETOPAUSE

The magnetopause seems to be the most suitable region for the application of the analytical expressions derived in this paper. Indeed, the plasma flow velocity changes here within the thin ($\sim R_E$) boundary layer dividing the magnetosheath from the plasmasheet or the plasma mantle. Moreover, the magnetopause proper, i.e. layer where the magnetic field sharply changes (on a distance much less than 1 R_E), divides the boundary layer from the magnetosheath. Accordingly, we can assume that changing the plasma flow velocity begins on the magnetopause; the first derivative of the plasma velocity jumps here. In this case our analytical solutions show that the surface wave resonates with velocity vortices in regions of varying speed are propagate along the magnetopause.

First, we shall consider the magnetopause on the flanks of the magnetosphere. We shall take the plasma parameters in the boundary layer (medium 1) and in the magnetosheath (medium 2) to be following: $V_{a2} = 40$ km/s, $C_{s2} = 100$ km/s, $V_{a1} = 750$ km/s, $C_{s1} = 800$ km/s, $\rho_2/\rho_1 = 100$, and the plasma speed to be changing from 400 km/s in medium 2 to zero on the inner edge of the boundary layer. We shall use a coordinate system moving with the magnetosheath plasma particles.

Then the dispersion equation will be expressed by formula (56). In this case:

$$C_{f1}^2 \gg \Omega_o^2, \quad C_{f2} > \Omega_o^2.$$

$$[(V'_{z1}/V_{z1})_o + U'_{xo}/\Omega_o][1-\Omega_o^2/C_{f2}^2]^{1/2} = k\rho_2/\rho_1 \quad (65)$$

The differential equation in this case is the Rayleigh equation (16). Unfortunately, a strict solution cannot be obtained by a simple method and we can only make rough estimates. Examining expression (65) and using experimental data on the plasma parameters, we see that

$$\mathrm{Re}\,(U'_{xo}/\Omega_o) \gg \mathrm{Re}\,(V'_{z1}/V_{z1})_o \sim k.$$

Then:

$$\Omega_o^2 = 1\bigg/\left(\left(\frac{k^2\rho_2}{U'_{xo}\rho_1}\right)^2 + 1/C_{f2}^2\right), \quad (66)$$

$$T^2 = (2\pi)^2\left(\left(\frac{\rho_2}{U'_{xo}\rho_1}\right)^2 + 1/(k^2C_{f2}^2)\right) \quad (67)$$

Using only the first term in this sum gives an oscillation period equal to 25 min for $U'_{xo} = 0.4\ \mathrm{s}^{-1}$. Since we are not going to deal with oscillations having much longer periods, we can examine only such oscillations as below:

$$k^2 \gg \left(\frac{U'_{xo}\rho_1}{C_{f2}\rho_2}\right)^2, \quad (68)$$

Then we have in (65) $1 \gg \Omega_o^2/C_{f2}^2$ and

$$\Omega_o = U'_{xo}/(k\rho_2/\rho_1 - (V'_{z1}/V_{z1})_o), \quad (69)$$

$$U_{ph} = \frac{U'_{xo}\rho_1}{k\rho_2}, \quad (70)$$

$$\gamma = -\pi\frac{\rho_1 U'_{xo} U''_x(z_c)}{\rho_2 k U'_x(z_c)}\exp(-2kz_c). \quad (71)$$

These expressions are very similar to formulae (57) and (61–62). However, here we have $\Omega^2 < C_{f2}^2$ and, therefore, we cannot assume that $U_{ph} = U_{x\max}$. The maximum growth rate will be for minimally possible wave numbers among all oscillations (68). Suppose

$$k = 2\frac{\rho_1 U'_{xo}}{\rho_2 C_{f2}}. \quad (72)$$

Then

$$U_{ph} = C_{f2}/2 \cong 50\,\mathrm{km/s}, \quad T = 30\,\mathrm{min},$$

$$\gamma = 0.1\ \mathrm{s}^{-1}, \quad \lambda = 9\times 10^4\,\mathrm{km}.$$

In the coordinate system at rest relative to the Earth, we have: $U_{ph} = 350$ km/s, $T = 4$ min. Oscillations, generated in this part of the magnetopause, can show up in the polar ionosphere as mesoscale auroral structures near the poleward boundary of the cusp.

Now we examine that part of the magnetopause close to the subsolar point, for example, regions located in the equatorial plane within +3 h of the noon meridian. The magnetospheric magnetic field here is orthogonal to the plasma flow as well. We assume the plasma parameters to be the following: $V_{a2} = 120$ km/s, $C_{s2} = 170$ km/s, $V_{a1} = 400$ km/s, $C_{s1} = 800$ km/s, $\rho_2/\rho_1 = 10$ and plasma velocity to be changing from 200 km/s in the magnetosheath to zero on the inner edge of the boundary layer. The formulae remain the same and we obtain $U_{ph} = 100$ km/s, $T = 3$ min, $\gamma = 0.2\ \mathrm{s}^{-1}$, $\lambda = 2\times 10^4$ km. In the coordinate system at rest relative to the Earth, we have the same wave parameters.

These oscillations map into the polar ionosphere near the equatorward boundary of the cusp.

8. SUMMARY

The main objective of this paper is to reveal the mechanism of the Kelvin–Helmholtz instability in a compressible plasma with a magnetic field. It has been shown that the wave propagating on the surface of the maximum of the second derivative of the flow velocity has the greatest growth rate. The phase speed and growth rate of this wave depend mainly on conditions at the resonant points. There are four resonant points for the pressure variables and nine points for the transverse speed variables.

The main flow velocity at these resonant points differs from the phase speed of oscillations by certain quantities such as the Alfven speed, fast and slow magnetosonic speeds, and cusp resonance speed. This means that only those perturbations with these speed differences provide feedback at the resonant points. The Kelvin–Helmholtz instability is developed if the feedback is positive.

For the velocity profile with a strongly developed maximum of second derivative of the velocity (sharp elbow), analytical expressions for the phase speed, growth rate and wavenumber of the fastest growing unstable mode have been obtained. These expressions have been applied with some necessary simplifications to regions of the magnetosphere where sheared plasma flows are observed such as the magnetopause, and the boundary between the inner plasma sheet and the plasmasphere. Simple analytical expressions have been derived for parameters describing the plasma oscillations. These parameters evaluated for three above mentioned magnetospheric regions are presented in Table 1 .

Table 1.

Magnetospheric region	Period (minutes)	Wavelength (10^4 km)	Phase speed (km/s)	Growth rate (s^{-1})
Boundary between plasma sheet and plasma sphere	10	6	100	0.02
Boundary between plasma sheet and tail lobes	3	5.4	300	0.14
Equatorial magnetopause on the flanks	4	9	350	0.1
Equatorial magnetopause on 45° from noon meridian	3	2	100	0.2

A comparison between the calculated and observed parameters of plasma oscillations is practicable only for the magnetopause where experimental data are available. The following values are typical of magnetopause plasma oscillations: the period varies from 150 to 600, the phase speed is in the range 50–250 km/s, the growth rate is $\sim 0.02\ s^{-1}$ (Kivelson *et al.*, 1984; Lui *et al.*, 1987; Junginger and Baumjohann, 1988; Wolfe *et al.*, 1987; Potemra *et al.*, 1990; 1992; Sibeck *et al.*, 1990). We note a general agreement between the calculated and observed parameters of the plasma oscillations. Therefore, we conclude that our simple analytical expressions for the KHI give adequate results.

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